Rutgers Business School
Newark and New Brunswick

Advanced Probability 26:711:685:02	**Michael N. Katehakis**
Fall 2017	1070 1 WP
Room: TBA	Home page
Wednesdays 2:30-5:20	Office Hours: after class, and by appointment

Course Description

This course provides a rigorous introduction to probability theory. It covers probability spaces, random variables and distribution functions. Independence, Borel-Cantelli lemma, zero-one laws. expectation, sums of independent random variables, stopping times, Wald's equations, elementary renewal theorems. Laws of large numbers. Characteristic functions. Central limit problem; Lindeberg-Feller theorem, Law of the iterated logarithm. Conditional probability and expectation. Martingales and the Martingale Convergence Theorem. Introduction to concentration of measure, Cramer's theorem, and large deviations.

For students in the doctoral program only. Students in a masters program must seek permission from the Director of the doctoral program.

Course Materials:

◇ *A Course in Probability Theory* by K.L. Chung, 2001

◇ *Probability Essentials* by Jacod, J. and Protter, P. , Springer Science & Business Media, 2004.

◇ *Concentration of measure inequalities* by G. Lugosi.

◇ *Other Resources:*

- Laha, R. G. and Rohatgi, V. K. (1977). Probability Theory. Wiley
- Chow, Y. S. and Teicher, H. (1978). Probability Theory. Springer
- Dudley, R. M. (1989). Real Analysis and Probability. Wadsworth and Brooks/Cole
- Billingsley, P. (1968). Convergence of Probability Measures. Wiley
- Loève, M. (1977). Probability Theory I and II. Springer
- Ash, R. B. (1972). Real Analysis and Probability. Academic Press
- Rudin Walter (1976). Principles of Mathematical Analysis, 3-rd ed. McGraw-Hill.

I will provide additional reading materials during the course.

◇ Start with: A Course in Probability Theory.

Learning Goals and Objectives: This course will introduce students to some of the important probability concepts. Specific goals: There will be an introduction to the concepts and applications of those basic ideas that are considered to be most important for the practical analysis of management, listed below.

1. Probability spaces

2. Random variables and distribution functions
3. Independence
4. Borel-Cantelli lemma
5. Zero-one laws
6. Sums of independent random variables
7. Stopping times, Wald's equations
8. Elementary renewal theorems
9. Laws of large numbers
10. Characteristic functions
11. Central limit problem
12. Lindeberg-Feller theorem,
13. Law of the iterated logarithm
14. Conditional probability and expectation
15. Martingales and the Martingale Convergence Theorem
16. Introduction to concentration of measure
17. Cramer's theorem, and large deviations

Prerequisites:

Students in the class must be familiar with i) undergraduate-level probability and ii) a basic real analysis course, Rudin's is a sufficient reference.

Academic Integrity:
Students are responsible for understanding the RU Academic Integrity Policy. Students must sign the RU Honor Pledge. See business.rutgers.edu/ai for more details.

Teaching Method:
The course will be largely taught using class presentations. Class-related material (lecture notes, messages, etc.) will be posted. Additional material and demos may be shown in some classes. *Most of your work will take place outside the classroom, as you study, and apply the material to which you are introduced in class.*

Grading: Your grade[1] will be based on two midterm quizes (15% each), a comprehensive final exam (50%), and homework (20%). The exams will be closed-book but you'll be allowed to use a page or two of notes.

Homework Assignments are designed to help you learn the material discussed in class. In addition doing a thorough job on the homework assignments is the best preparation for the quiz and the final examination. There are three types of assignments: read, prepare, and hand in.

[1] Your final grade is not subject to negotiation. If you feel I have made an error, submit your written argument to me within one week of receiving your final grade. Clarify the precise error I made and provide all due supporting documentation. If I have made an error, I will gladly correct it. But I will adjust grades only if I have made an error.

- **Read:** When the assignment is to read some material, this reading is an important introduction to the topics to be discussed in class. I will make the assumption that you have done the reading before class and have understood much (but not necessarily all) of it. When the assignment is to read a problem, that problem will often be used in class to introduce new concepts.
- **Prepare:** Fully analyze the problem. Be ready to discuss it in class, with the numbers computed, etc. I will call on people, so please be ready.
- **Hand In:** The same as prepare, but you must turn in your analysis. All written assignments must be handed in at the beginning of class on the day they are due, and so you will probably want to make a copy of your assignment for reference during class. All written assignments will be graded. These assignments should be submitted in typed form using a word processor. Please write your name, RUID and email on all homework submitted. Team work on this homework is not allowed. Unless a documented reason is produced for unusual circumstances, late submissions will not be accepted more than a week late.

Class Participation and Conduct. Your class participation will be evaluated subjectively, but will rely upon measures of punctuality, attendance, familiarity with the required readings, relevance and insight reflected in classroom questions, and commentary. Your class participation will be judged by what you add to the class environment, regardless of your technical background. Although several lectures will be didactic, we will rely heavily upon interactive discussion within the class. Students will be expected to be familiar with the readings, even though they might not understand all of the material in advance. In general, questions and comments are encouraged. Comments should be limited to the important aspects of earlier points made, and reflect knowledge of the readings. You may called on to answer questions about the homework or classroom discussion. Your classroom participation evaluation is based on the extent to which you contribute to the learning environment. However, correcting a mistake of the professor and asking what appear to be "dumb questions" about what is being covered are positive contributions. In the case of so-called "dumb questions", very often half of the class will have the same questions in mind and are relieved to have them asked.

Tentative Course Outline.

Sections labeled C refer to Chung, sections labeled JP refer to Jacod, J. and Protter,.

◇ Weeks 1-3: Basic Concepts
Sections C-1.1 through C-3.3 of chapter 3, and JP-8 to JP-10

- Distribution functions
  - Monotone functions
  - Distribution functions
  - Absolutely continuous and singular distributions
- Probabilities and Measures
  - Classes of Sets
  - Probability measures and their distribution functions
- Random variables Expectation Independence
  - Expectations
  - Independence

- ◇ Week 4: Convergence concepts
  Sections C-4.1 through C-4.5
  - Modes of convergence
  - Zero-One Laws, Borel-Cantelli Lemma
- ◇ Week 5: *Quiz #1*
- ◇ Weeks 6-7 Random Series,
  Sections C-5.1 through C-5.5 & JP-15, JP-20

  Law of large numbers

  Renewal theorem

  The Kolmogorov Three Series Theorem
- ◇ Weeks 8-9:
  Sections C-6.1 through C-7.5 & JP-13, JP-14, JP-21

  Characteristic function

  Central limit theorem

  Law of the iterated logarithm

  Lindeberg-Feller Central limit theorem
- ◇ Week 10: *Quiz #2*
- ◇ Week 11: Concentration-of-measure inequalities
  Sections from *Concentration of measure inequalities* by G. Lugosi and class notes.

  Hoeffding's inequality

  Bernstein's inequality

  Efron-Stein inequality

  Chernoff bounds

  Dvoretzky-Kiefer-Wolfowitz inequality
- ◇ Weeks 12-13 Martingales Examples, Basic Properties.
  Chapters 24 though 27 of JP
- ◇ Week 14: *Final exam*